
Data Structures and Algorithms with Object-Oriented Design Patterns in C#

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Contents

Preface	xi
1 Introduction	1
1.1 What This Book Is About	1
1.2 Object-Oriented Design	1
1.3 Object Hierarchies and Design Patterns	2
1.4 The Features of C# You Need to Know	3
1.5 How This Book Is Organized	5
2 Algorithm Analysis	7
2.1 A Detailed Model of the Computer	8
2.2 A Simplified Model of the Computer	21
Exercises	30
Programming Projects	32
3 Asymptotic Notation	33
3.1 An Asymptotic Upper Bound—Big Oh	33
3.2 An Asymptotic Lower Bound—Omega	43
3.3 More Notation—Theta and Little Oh	46
3.4 Asymptotic Analysis of Algorithms	46
Exercises	58
Programming Projects	61
4 Foundational Data Structures	63
4.1 Arrays	63
4.2 Multi-Dimensional Arrays	69
4.3 Singly-Linked Lists	76
Exercises	86
Programming Projects	87
5 Data Types and Abstraction	89
5.1 Abstract Data Types	89
5.2 Design Patterns	90
Exercises	110
Programming Projects	112

6	Stacks, Queues, and Deques	113
6.1	Stacks	113
6.2	Queues	126
6.3	Deques	134
	Exercises	141
	Programming Projects	143
7	Ordered Lists and Sorted Lists	145
7.1	Ordered Lists	146
7.2	Sorted Lists	168
	Exercises	179
	Programming Projects	180
8	Hashing, Hash Tables, and Scatter Tables	183
8.1	Hashing—The Basic Idea	183
8.2	Hashing Methods	186
8.3	Hash Function Implementations	190
8.4	Hash Tables	198
8.5	Scatter Tables	204
8.6	Scatter Table using Open Addressing	213
8.7	Applications	225
	Exercises	228
	Programming Projects	230
9	Trees	231
9.1	Basics	232
9.2	N -ary Trees	235
9.3	Binary Trees	238
9.4	Tree Traversals	239
9.5	Expression Trees	241
9.6	Implementing Trees	243
	Exercises	269
	Programming Projects	271
10	Search Trees	273
10.1	Basics	273
10.2	Searching a Search Tree	275
10.3	Average Case Analysis	276
10.4	Implementing Search Trees	282
10.5	AVL Search Trees	287
10.6	M -Way Search Trees	298
10.7	B-Trees	308
10.8	Applications	319
	Exercises	319
	Programming Projects	322

11 Heaps and Priority Queues	325
11.1 Basics	325
11.2 Binary Heaps	327
11.3 Leftist Heaps	337
11.4 Binomial Queues	343
11.5 Applications	358
Exercises	361
Programming Projects	363
12 Sets, Multisets, and Partitions	365
12.1 Basics	366
12.2 Array and Bit-Vector Sets	368
12.3 Multisets	374
12.4 Partitions	382
12.5 Applications	393
Exercises	395
Programming Projects	396
13 Garbage Collection	399
13.1 What is Garbage?	400
13.2 Reference Counting Garbage Collection	402
13.3 Mark-and-Sweep Garbage Collection	406
13.4 Stop-and-Copy Garbage Collection	408
13.5 Mark-and-Compact Garbage Collection	410
Exercises	413
Programming Projects	414
14 Algorithmic Patterns and Problem Solvers	417
14.1 Brute-Force and Greedy Algorithms	417
14.2 Backtracking Algorithms	420
14.3 Top-Down Algorithms: Divide-and-Conquer	428
14.4 Bottom-Up Algorithms: Dynamic Programming	437
14.5 Randomized Algorithms	444
Exercises	454
Programming Projects	456
15 Sorting Algorithms and Sorters	459
15.1 Basics	459
15.2 Sorting and Sorters	460
15.3 Insertion Sorting	461
15.4 Exchange Sorting	466
15.5 Selection Sorting	477
15.6 Merge Sorting	485
15.7 A Lower Bound on Sorting	489
15.8 Distribution Sorting	491
15.9 Performance Data	497
Exercises	499
Programming Projects	501

16	Graphs and Graph Algorithms	503
16.1	Basics	504
16.2	Implementing Graphs	511
16.3	Graph Traversals	520
16.4	Shortest-Path Algorithms	533
16.5	Minimum-Cost Spanning Trees	543
16.6	Application: Critical Path Analysis	551
	Exercises	555
	Programming Projects	559
A	C# and Object-Oriented Programming	561
A.1	Variables	561
A.2	Parameter Passing	563
A.3	Objects and Classes	567
A.4	Nested Classes	571
A.5	Inheritance and Polymorphism	572
A.6	Exceptions	580
B	Class Hierarchy Diagrams	583
C	Character Codes	585
	Index	591

Preface

This book was motivated by my experience in teaching the course *EECE 250: Algorithms and Data Structures* in the Computer Engineering program at the University of Waterloo. I have observed that the advent of *object-oriented methods* and the emergence of object-oriented *design patterns* has led to a profound change in the pedagogy of data structures and algorithms. The successful application of these techniques gives rise to a kind of cognitive unification: Ideas that are disparate and apparently unrelated seem to come together when the appropriate design patterns and abstractions are used.

This paradigm shift is both evolutionary and revolutionary. On the one hand, the knowledge base grows incrementally as programmers and researchers invent new algorithms and data structures. On the other hand, the proper use of object-oriented techniques requires a fundamental change in the way the programs are designed and implemented. Programmers who are well schooled in the procedural ways often find the leap to objects to be a difficult one.

❖ Goals

The primary goal of this book is to promote object-oriented design using C# and to illustrate the use of the emerging *object-oriented design patterns*. Experienced object-oriented programmers find that certain ways of doing things work best and that these ways occur over and over again. The book shows how these patterns are used to create good software designs. In particular, the following design patterns are used throughout the text: *singleton*, *container*, *enumeration*, *adapter* and *visitor*.

Virtually all of the data structures are presented in the context of a *single, unified, polymorphic class hierarchy*. This framework clearly shows the *relationships* between data structures and it illustrates how polymorphism and inheritance can be used effectively. In addition, *algorithmic abstraction* is used extensively when presenting classes of algorithms. By using algorithmic abstraction, it is possible to describe a generic algorithm without having to worry about the details of a particular concrete realization of that algorithm.

A secondary goal of the book is to present mathematical tools *just in time*. Analysis techniques and proofs are presented as needed and in the proper context. In the past when the topics in this book were taught at the graduate level, an author could rely on students having the needed background in mathematics. However, because the book is targeted for second- and third-year students, it is necessary to fill in the background as needed. To the extent possible without compromising correctness, the presentation fosters intuitive understanding of the concepts rather than mathematical rigor.

❖ Approach

One cannot learn to program just by reading a book. It is a skill that must be developed by practice. Nevertheless, the best practitioners study the works of others and incorporate their observations into their own practice. I firmly believe that after learning the rudiments of program writing, students should be exposed to examples of complex, yet well-designed program artifacts so that they can learn about the designing good software.

Consequently, this book presents the various data structures and algorithms as complete C# program fragments. All the program fragments presented in this book have been extracted automatically from the source code files of working and tested programs. It has been my experience that by developing the proper abstractions, it is possible to present the concepts as fully functional programs without resorting to *pseudo-code* or to hand-waving.

❖ Outline

This book presents material identified in the *Computing Curricula 1991* report of the ACM/IEEE-CS Joint Curriculum Task Force[45]. The book specifically addresses the following *knowledge units*: AL1: Basic Data structures, AL2: Abstract Data Types, AL3: Recursive Algorithms, AL4: Complexity Analysis, AL6: Sorting and Searching, and AL8: Problem-Solving Strategies. The breadth and depth of coverage is typical of what should appear in the second or third year of an undergraduate program in computer science/computer engineering.

In order to analyze a program, it is necessary to develop a model of the computer. Chapter 2 develops several models and illustrates with examples how these models predict performance. Both average-case and worst-case analyses of running time are considered. Recursive algorithms are discussed and it is shown how to solve a recurrence using repeated substitution. This chapter also reviews arithmetic and geometric series summations, Horner's rule and the properties of harmonic numbers.

Chapter 3 introduces asymptotic (big-oh) notation and shows by comparing with Chapter 2 that the results of asymptotic analysis are consistent with models of higher fidelity. In addition to $O(\cdot)$, this chapter also covers other asymptotic notations ($\Omega(\cdot)$, $\Theta(\cdot)$, and $o(\cdot)$) and develops the asymptotic properties of polynomials and logarithms.

Chapter 4 introduces the *foundational data structures*—the array and the linked list. Virtually all the data structures in the rest of the book can be implemented using either one of these foundational structures. This chapter also covers multi-dimensional arrays and matrices.

Chapter 5 deals with abstraction and data types. It presents the recurring design patterns used throughout the text as well a unifying framework for the data structures presented in the subsequent chapters. In particular, all of the data structures are viewed as *abstract containers*.

Chapter 6 discusses stacks, queues, and deques. This chapter presents implementations based on both foundational data structures (arrays and linked lists). Applications for stacks and queues are presented.

Chapter 7 covers ordered lists, both sorted and unsorted. In this chapter, a list is viewed as a *searchable container*. Again several applications of lists are presented.

Chapter 8 introduces hashing and the notion of a hash table. This chapter addresses the design of hashing functions for the various basic data types as well as

for the abstract data types described in Chapter 5. Both scatter tables and hash tables are covered in depth and analytical performance results are derived.

Chapter 9 introduces trees and describes their many forms. Both depth-first and breadth-first tree traversals are presented. Completely generic traversal algorithms based on the use of the *visitor* design pattern are presented, thereby illustrating the power of *algorithmic abstraction*. This chapter also shows how trees are used to represent mathematical expressions and illustrates the relationships between traversals and the various expression notations (prefix, infix, and postfix).

Chapter 10 addresses trees as *searchable containers*. Again, the power of *algorithmic abstraction* is demonstrated by showing the relationships between simple algorithms and balancing algorithms. This chapter also presents average case performance analyses and illustrates the solution of recurrences by telescoping.

Chapter 11 presents several priority queue implementations, including binary heaps, leftist heaps, and binomial queues. In particular this chapter illustrates how a more complicated data structure (leftist heap) extends an existing one (tree). Discrete-event simulation is presented as an application of priority queues.

Chapter 12 covers sets and multisets. Also covered are partitions and disjoint set algorithms. The latter topic illustrates again the use of algorithmic abstraction.

Garbage collection is discussed in Chapter 13. This is a topic that is not found often in texts of this sort. However, because the C# language relies on garbage collection, it is important to understand how it works and how it affects the running times of programs.

Chapter 14 surveys a number of algorithm design techniques. Included are brute-force and greedy algorithms, backtracking algorithms (including branch-and-bound), divide-and-conquer algorithms, and dynamic programming. An object-oriented approach based on the notion of an *abstract solution space* and an *abstract solver* unifies much of the discussion. This chapter also covers briefly random number generators, Monte Carlo methods, and simulated annealing.

Chapter 15 covers the major sorting algorithms in an object-oriented style based on the notion of an *abstract sorter*. Using the abstract sorter illustrates the relationships between the various classes of sorting algorithm and demonstrates the use of algorithmic abstractions.

Finally, Chapter 16 presents an overview of graphs and graph algorithms. Both depth-first and breadth-first graph traversals are presented. Topological sort is viewed as yet another special kind of traversal. Generic traversal algorithms based on the *visitor* design pattern are presented, once more illustrating *algorithmic abstraction*. This chapter also covers various shortest path algorithms and minimum-spanning-tree algorithms.

At the end of each chapter is a set of exercises and a set of programming projects. The exercises are designed to consolidate the concepts presented in the text. The programming projects generally require the student to extend the implementation given in the text.

❖ Suggested Course Outline

This text may be used in either a one semester or a two semester course. The course which I teach at Waterloo is a one-semester course that comprises 36 lecture hours on the following topics:

1. Review of the fundamentals of programming in C# and an overview of object-oriented programming with C#. (Appendix A). [4 lecture hours].
2. Models of the computer, algorithm analysis, and asymptotic notation (Chapters 2 and 3). [4 lecture hours].
3. Foundational data structures, abstraction, and abstract data types (Chapters 4 and 5). [4 lecture hours].
4. Stacks, queues, ordered lists, and sorted lists (Chapters 6 and 7). [3 lecture hours].
5. Hashing, hash tables, and scatter tables (Chapter 8). [3 lecture hours].
6. Trees and search trees (Chapters 9 and 10). [6 lecture hours].
7. Heaps and priority queues (Chapter 11). [3 lecture hours].
8. Algorithm design techniques (Chapter 14). [3 lecture hours].
9. Sorting algorithms and sorters (Chapter 15). [3 lecture hours].
10. Graphs and graph algorithms (Chapter 16). [3 lecture hours].

Depending on the background of students, a course instructor may find it necessary to review features of the C# language. For example, an understanding of *inner classes* is required for the implementation of *enumerations*. Similarly, students need to understand the workings of *classes*, *interfaces*, and *inheritance* in order to understand the unifying class hierarchy discussed in Chapter 5.

❖ Online Course Materials

Additional material supporting this book can be found on the world-wide web at the URL:

<http://www.pads.uwaterloo.ca/Bruno.Preiss/books/opus6>

In particular, you will find there the source code for all the program fragments in this book as well as an errata list.

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